Representing numbers using computers

1) Convert the following to base-10. Decimal values can be expressed as rational numbers.
   1001 1101
   1101 0010
   1010 0011 .   0011 1111
   1110 0001 .   1011 0001

   . 10011 x $(2^3)_{10}$

2) Assume you are using a base-10 floating point system with a 3 digit normalized mantissa.
   a. What is the smallest positive value that can be expressed with an exponent of 3_{10}?
   b. What is the spacing (ϵ_{10}) between values in this decade?
   c. How does this compare to the spacing (ϵ_2) of a base-2 floating point system with a 3 (binary) digit normalized mantissa and an exponent of 11_2?

3) By default, Matlab uses the IEEE 64-bit floating point standard with a 52-bit mantissa. Compute the result of $y = \sqrt{x+4} - 2$, where $x = .721273 \times 10^{-3}$ using Matlab. Keep 6 significant figures.
   a. Simulate a lower-precision computer: calculate the same function step-by-step keeping 3 significant digits at each step. You can use a calculator for each step – just round to 3 significant figures. Where does the catastrophic cancellation occur?
   b. What is the relative error?
   c. Reformulate the function to avoid catastrophic cancellation and recalculate using the same (6-digit) precision. What is the relative error?
   d. Use the Loss of Precision Theorem to estimate how many significant binary bits are lost in this calculation.